

February 6, 1890.

Mr. JOHN EVANS, D.C.L., Treasurer and Vice-President, in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read :—

- I. "A New Theory of Colour-blindness and Colour-perception." By F. W. EDRIDGE-GREEN, M.D. Communicated by Dr. LAUDER BRUNTON, F.R.S. Received January 28, 1890.
- II. "Memoir on the Symmetrical Functions of the Roots of Systems of Equations." By Major P. A. MACMAHON, Royal Artillery. Communicated by Professor GREENHILL, F.R.S. Received January 30, 1890.

(Abstract.)

The object of the present memoir is the extension to systems of algebraical quantities of the new theory of symmetric functions which has been developed by the author in regard to a single system in Volume 11 and succeeding volumes of the 'American Journal of Mathematics.' In the theory of the single system the conceptions and symbolism are to a large extent arithmetical, and are based upon the properties of single integral numbers and their partitions into single integral parts. In this sense the theory may be regarded as being unipartite. In the present generalisation to the case of m systems of quantities the fundamental ideas proceed, not from a single number, but from a collection of m single numbers. In regard to number, weight, degree, part, and suffix, the collection of m numbers invariably replaces the single number of the theory of the single system. In this view the theory of the m systems is m -partite.

The quantities, to which the symmetric functions relate, may be regarded as the solutions common to m non-homogeneous equations each in m variables. Schlafli, in the 'Vienna Transactions' (*Denkschriften*) for 1852, added another linear non-homogeneous equation in m variables, and then forming the eliminant of the $m + 1$ equations,

thereby obtained an identity which is fundamental in the subject. This identity involves those symmetric functions which are here termed fundamental, and marks the starting point of the present investigation.

The memoir is divided into sixteen sections. In § 2 a preliminary algebraic theory is given, and then in § 3 is commenced the theory of the differential operations. A prominent feature presents itself in the very interesting correspondence between the algebras of quantity and differential operation.

In § 4 is discussed the theory of three identities, formed similarly to the fundamental identity alluded to above, and such that the quantities involved are related in a particular manner. The theory of differential operation proceeds collaterally with that of quantity. The succeeding four sections, § 5–§ 8, are devoted to the results which flow in a direct manner from this discussion. In particular, three distinct laws of symmetry are established, large generalisations of those established by the author in the ‘American Journal of Mathematics’ (*loc. cit.*). Of these the first two are of importance, and are examined in detail. A leading idea in these theorems, as in the whole investigation, is the “separation” of a partition; the separation bears the same relation to the partition as the partition to the number or collection of numbers. The first law of symmetry appears to be of cardinal rank in symmetrical algebra. It involves, at sight, a law of expressibility in the theory of separations which is of a general character. It demonstrates at once the possibility of forming a pair of symmetrical tables of symmetric functions in connexion with every partition of every collection of m numbers (regard being paid as well to order as to magnitude). The necessary tables for the bipartite theory (*i.e.*, of two systems) as far as the weight, four inclusive, are exhibited in § 14. An extension of the Vandermonde-Waring law for the expression of the sums of the powers of the roots of an equation by means of the coefficients is generalised, in a single formula, from two points of view in § 6. In § 9 and § 10 the decomposition is effected of the operations previously encountered in § 3. The linear weight operations are found to break up into as many linear partition operations as the weight possesses partitions. An important theorem is reached when it is established that the annihilation of a symmetric function by a linear weight operation necessitates annihilation by each partition operation of the same weight. The weight operations of higher orders, partially examined in § 3, which may be termed “obliterating,” from their characteristic property, break up similarly into partition operations which possess an obliterating property in regard to products of symmetric functions. In this manner all the differential operations of § 3 are adapted for use in the theory of *separations*, as distinct from the theory ordinarily

considered, which is in fact that of fundamental or single-unitary symmetric functions.

These partition operations possess an algebra also in exact correspondence with the algebra of quantity.

In § 10 the partition obliterating operations are applied to the theory of multiplication.

In § 12 a transformation is established by means of which functions of differences can, with special exceptions, be converted into non-single-unitary symmetric functions. This theorem is the analogue of the transformation of the theory of invariants first given by the author in Vol. 6 of the 'American Journal of Mathematics.'

§ 15 proves a useful law to which the tabular numbers are subject, connected with the idea of grouping the separations together in a particular manner.

In conclusion the memoir consolidates and largely generalises the author's recent researches alluded to above at the beginning of the Abstract.

Presents, February 6, 1890.

Transactions.

Brisbane :—Royal Geographical Society of Australia (Queensland Branch). Proceedings and Transactions. Vol. IV. 8vo. *Brisbane* 1889. The Society.

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Copenhagen :—Académie Royale. Bulletin. 1889. No. 2. 8vo. *Copenhague*; Mémoires (Classe des Sciences). Vol. V. Nos. 1-2. 4to. *Copenhague* 1889. The Academy.

Córdoba :—Academia Nacional de Ciencias. Boletin. Tomo XI. Entrega 3. 8vo. *Buenos Aires* 1888. The Academy.

Cracow :—Académie des Sciences. Bulletin International. Comptes Rendus des Séances de l'Année 1889. Nos. 5-10. 8vo. *Cracovie* 1889. The Academy.

Dresden :—Verein für Erdkunde. Jubiläumsschrift. Litteratur der Landes- und Volkskunde des Königreichs Sachsen. Bearbeitet von Pa Emil Richter. 8vo. *Dresden* 1889. The Verein.

Dublin :—Royal Historical and Archaeological Association of Ireland. Journal. Vol. IX. No. 80. 8vo. *Dublin* 1889. The Association.

Edinburgh :—Royal Physical Society. Proceedings. Vol. X. Part 1. 8vo. *Edinburgh* 1889. The Society.